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FAST ARRAY ALGORITHMS FOR FILTERING OF MARKOVIAN JUMP LINEAR SYSTEMS WITH STRUCTURED TIME-VARIANT PARAMETERS

ALGORITMOS ARRAY RÁPIDOS PARA FILTRAGEM DE SISTEMAS LINEARES SUJEITOS A SALTOS MARKOVIANOS COM VARIAÇÃO ESTRUTURADA DOS PARÂMETROS NO TEMPO

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Abstract: In this paper were developed fast array algorithms for the linear minimum mean square error estimator for a class of Markovian jump linear systems with structured time-variant parameters. The fast array algorithms for systems with structured time-variant parameters arises as an alternative to calculate this type algorithm for some variation in the time of the parameters. Numerical example to show the advantage of using fast array algorithm to filter this class of systems are provided.

Keywords: Fast Array Algorithm. Structured Variant-time. Filtering. Markovian Systems.

Resumo: Neste artigo foram desenvolvidos algoritmos *array* rápidos para o estimador linear mínimo médio quadrático para uma classe de sistemas lineares sujeitos a saltos Markovianos com variação estruturada dos parâmetros no tempo. Os algoritmos *array* rápidos para sistemas com variação estruturada dos parâmetros no tempo surgem como uma alternativa para o cálculo deste tipo de algoritmo para alguma variação dos parâmetros no tempo. Exemplos numéricos que mostram as vantagens de usar este tipo de algoritmo *array* rápido para esta classe de sistemas são fornecidos.

Palavras-chave: Algoritmos *Array* Rápidos. Variação Estruturada. Filtragem. Sistemas Markovianos.

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DOI: 10.21575/25254782rmetg2021vol6n41626

1 Introduction

Filtering for Markovian jump linear systems (MJLS) has been a topic widely studied in the literature (ACKERSON; FU, 1970), (CHANG; ATHANS, 1978), (COSTA, 1994), (COSTA; GUERRA, 2002), (GONÇALVES; FIORAVANTI; GEROMEL, 2010), (COSTA; BENITES, 2011), (MATEI; BARAS, 2012), (YANG et al., 2013), (LI; JIA, 2013), (YIN et al., 2014) and (ZHONG et al., 2014). In (COSTA; GUERRA, 2002) was developed the linear minimum mean square error estimator (LMMSE) for MJLS, based in the recursive Riccati equation, this type of filter is very useful in online applications, because its recursive nature. However some numerical problems has been detected that make the recursion of the Riccati equation unstable numerically, as for example, round-off error that make the Riccati variable non-Hermitian or indefinite (MORF; KAILATH, 1974), (ZORZI, 2017), (LEVY; ZORZI, 2016). To solve this problem has been proposed alternative algorithms for compute these filters, for example, square-root array algorithms (MORF; KAILATH, 1975), (TERRA; ISHIHARA; JESUS, 2009) and fast array algorithms (MORF; KAILATH, 1974), (SAYED; KAILATH, 1994), (HASSIBI; KAILATH; SAYED, 1999), (TERRA; ISHIHARA; JESUS, 2012). This methods has been applied in the literature as a resource to alleviate computational problems associated with the Riccati equation.

The fast array algorithms have computational advantages when compared to Riccati equation, for example, reduce the dynamic range in fixed-point implementations and assure better numerical conditions and computational cost. These were originally applied for filters, Kalman-type, in the space-state systems, in two situations: to invariant-time parameters (MORF; KAILATH, 1974) and to a class of structured time-variant parameters (SAYED; KAILATH, 1994). For this last class of systems, the fast array algorithms are also called extended Chandrasekhar recursions. The fast array algorithms for systems with time-variant parameters have not yet been developed in literature, therefore these algorithms for systems with structured time-variant parameters arises as an alternative to calculate this type algorithm for some variation in the time of the parameters. These algorithms propagate the factor M_i that is defined via relation $\delta P_i = P_{i+1} - \Psi_i P_i \Psi_i^T = M_i S_i M_i^T$, where the Ψ_i are convenient time-variant matrices. The fast array algorithms to a class of structured time-variant parameters has not yet been developed in filters for MJLS. This paper intends to develop this method for the LMMSE.

In order to show how effective are the algorithms proposed two numerical examples are presented: in the first, singular values are calculated through fixed- and floating-point computations based on Riccati equations and fast array algorithms; in the second, we show the evaluation of matrix M_i. We show also simulation results on the application of the proposed fast array algorithm for MJLS in the recursive least-square (RLS) problem in adaptive filtering.

This paper is organized as follows: Section 2 presents the LMMSE proposed in (COSTA; GUERRA, 2002) for MJLS; Section 3 presents the structured time-variant MJLS; Section 4 develops the fast array algorithm for filtering of structured time-variant MJLS; and Section 5 provides a comparative example among the numerical performance of the Riccati equation and fast array algorithms to compute the LMMSE for MJLS based on application to the RLS problem.

2 Preliminaries

In this section, we introduced LMMSE for MJLS. Let the following discrete-time MJLS

$$\begin{aligned} x_{i+1} &= F_{i,\Theta_i} x_i + G_{i,\Theta_i} u_i, \quad i = 0, 1 \ldots \\ y_i &= H_{i,\Theta_i} x_i + D_{i,\Theta_i} v_i. \end{aligned} \tag{1}$$

where x_i is the $\Re^n$ - valued state, u_i is the $\Re^p$ - random state disturbance, y_i is the $\Re^m$ - valued output sequence, and v_i is the $\Re^q$ - random output disturbance; $\{\Theta_i\}$ is a discrete-time Markov chain with finite state-space, where $\Theta_i \in \{1, \ldots, N\}$, and transition probability matrix $P = [p_{jk}]$. We set $\pi_{i,j} := P(\Theta_i = j)$; $F_{i,k} \in \Re^{n \times n}, G_{i,k} \in \Re^{n \times q_1}, H_{i,k} \in \Re^{m \times n}$, and $D_{i,k} \in \Re^{m \times q_2}$ $(k = 1, \ldots, N)$ are matrices which varying in time and in virtue of the Markov chain. The random disturbances $\{u_i\}$ and $\{v_i\}$ are null mean second-order, independent wide sense stationary sequences mutually independent with covariance matrices equal to identity; $x_0 1_{\{\Theta_0 = k\}}, k = 1, \ldots, N$, are random vectors with $\mathbb{E}\left\{x_0 1_{\{\Theta_0 = k\}}\right\} = \mu_k$ (where $1_{\{\cdot\}}$ denotes Dirac measure) and $\mathbb{E}\left\{x_0 x_0^T 1_{\{\Theta_0 = k\}}\right\} = V_k$; x_0, $\{\Theta_i\}$, $\{u_i\}$ and $\{v_i\}$ are independent.

The estimates are performed considering an augmented state variable

$$z_i := \left[\begin{array}{ccc} z_{i,1}^T & \ldots & z_{i,N}^T \end{array} \right]^T \in \Re^{Nn}, \tag{2}$$

$$z_{i,k} := x_i 1_{\{\Theta(i) = k\}} \in \Re^n. \tag{3}$$

Following (COSTA; GUERRA, 2002), the augmented model of (1) is written as

$$\begin{aligned} z_{i+1} &= \mathscr{F}_i z_i + \psi_i, \quad i = 0, 1 \ldots \\ y_i &= \mathscr{H}_i z_i + \varphi_i \end{aligned} \tag{4}$$

DOI: 10.21575/25254782rmetg2021vol6n41626

where the parameter matrices are given by

$$\mathscr{F}_i := \begin{bmatrix} p_{11}F_{i,1} & \dots & p_{N1}F_{i,N} \\ \vdots & \ddots & \vdots \\ p_{1N}F_{i,1} & \dots & p_{NN}F_{i,N} \end{bmatrix} \in \Re^{NnXNn}, \;\; \mathscr{H}_i := \begin{bmatrix} H_{i,1} & \dots & H_{i,N} \end{bmatrix} \in \Re^{mXNn}.$$

The random state and output disturbance variables, ψ_i and φ_i, are given by

$$\psi_i := \mathscr{M}_{i+1} z_i + \vartheta_i, \quad \varphi_i := D_{i,\Theta_i} v_i, \tag{5}$$

where $\mathrm{M}_{i+1} := \begin{bmatrix} \mathscr{M}_{i+1,1} \\ \vdots \\ \mathscr{M}_{i+1,N} \end{bmatrix}, \mathscr{M}_{i+1,k} := \begin{bmatrix} (1_{\{\Theta_{i+1}=k\}} - p_{1k})F_{i,1} \\ \vdots \\ (1_{\{\Theta_{i+1}=k\}} - p_{Nk})F_{i,N} \end{bmatrix}^T,$

$\vartheta_i := \begin{bmatrix} 1_{\{\Theta_{i+1}=1\}} G_{i,\Theta_i} u_i \\ \vdots \\ 1_{\{\Theta_{i+1}=N\}} G_{i,\Theta_i} u_i \end{bmatrix}.$

The LMMSE developed in (COSTA; GUERRA, 2002) is given by

$$\begin{aligned} \widehat{x}_{i|i} &= \sum_{j=1}^{N} \widehat{z}_{i,j|i}, && (6) \\ \widehat{z}_{i|i} &= \widehat{z}_{i|i-1} + Z_{i|i-1} \mathscr{H}_i^T (\mathscr{H}_i Z_{i|i-1} \mathscr{H}_i^T + R_i)^{-1} (y_i - \mathscr{H}_i \widehat{z}_{i|i-1}), && (7) \\ \widehat{z}_{i|i-1} &= \mathscr{F}_i \widehat{z}_{i-1|i-1}, \; \hat{z}_{0|-1} = \mathbb{E}(z_0) = \begin{bmatrix} \mu_1^T & \dots & \mu_N^T \end{bmatrix}^T, && (8) \\ \tilde{Z}_{i+1|i} &= \mathscr{F}_i \tilde{Z}_{i|i-1} \mathscr{F}_i^T - \mathscr{F}_i \tilde{Z}_{i|i-1} \mathscr{H}_i^T (\mathscr{H}_i \tilde{Z}_{i|i-1} \mathscr{H}_i^T + R_i)^{-1} \mathscr{H}_i \tilde{Z}_{i|i-1} \mathscr{F}_i^T + W_i, \\ \tilde{Z}_{0|-1} &= \Pi_0, && (9) \end{aligned}$$

where

$$\begin{aligned} R_i &= \mathscr{D}_i \mathscr{D}_i^T, && (10) \\ \mathscr{D}_i &= \begin{bmatrix} D_{i,1} \pi_{i,1}^{1/2} & \cdots & D_{i,N} \pi_{i,N}^{1/2} \end{bmatrix}, && (11) \\ \pi_{i,k} &= \mathscr{P}\big(\theta_i = k\big), && (12) \\ W_i &= diag\big[Z_{i+1,k}\big] - \mathscr{F}_i \Big(diag\big[Z_{i,k}\big]\Big) \mathscr{F}_i^T, && (13) \end{aligned}$$

where $Z_{i,k} \geq 0, k = 1, ..., N$ are given by the recursive equation

$$Z_{i+1,k} = \sum_{j=1}^{N} p_{jk} F_{i,j} Z_{i,j} F_{i,j}^T + \sum_{j=1}^{N} \pi_{i,j} p_{jk} G_{i,j} G_{i,j}^T, \;\; Z_{0,k} = \overline{V}_k. \tag{14}$$

DOI: 10.21575/25254782rmetg2021vol6n41626

3 MJLS with Structured Time-Variant Parameters

In this section, we developed the MJLS with structured time-variant parameters. Consider the parameters $F_{i,j}$, $G_{i,j}$ and $H_{i,j}$ of the Markovian jump linear system (1) as structured time-variant parameters if there exists matrices $\overline{\Psi}_i \in \Re^{n\times n}$ such that the parameters vary according to the following rules

$$\begin{aligned} F_{i+1,j}\overline{\Psi}_i &:= \overline{\Psi}_{i+1}F_{i,j}, \\ \overline{\Psi}_{i+1}G_{i,j} &:= G_{i+1,j}, \\ H_{i,j} &:= H_{i+1,j}\overline{\Psi}_i, \quad j,k = 1,2,...,N. \end{aligned} \tag{15}$$

The parameters $\mathscr{F}_i$ and $\mathscr{H}_i$ of the augmented model (4) vary according to the following rules if there exists matrices

$$\Psi_i := diag[\overline{\Psi}_i] = \begin{bmatrix} \overline{\Psi}_i & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \overline{\Psi}_i \end{bmatrix} \in \Re^{Nn\times Nn}, \tag{16}$$

such that

$$\begin{aligned} \mathscr{F}_{i+1}\Psi_i &:= \Psi_{i+1}\mathscr{F}_i, \\ \mathscr{H}_i &:= \mathscr{H}_{i+1}\Psi_i. \end{aligned} \tag{17}$$

Remark 3.1 *It is clear that $\overline{\Psi}_i = I_n$ satisfy constant-parameter in (15) and (17).*

4 Fast Array Algorithms for Filtering of Markovian Jump Linear Systems with Structured Time-Variant Parameters

In this section, the fast array algorithms for filtering of MJLS with structured time-variant parameters are presented. Details of the algorithms proofs can be found in (ANDRADE, 2015). We presented now a fast array algorithm to compute Lyapunov equation (14) as follow.

Algorithm 4.1 *(Fast Array Algorithm for Lyapunov Equation)*

Step 0 - *Initial conditions:*

$$Z_{0,j} := \overline{V}_j,$$

DOI: 10.21575/25254782rmetg2021vol6n41626

$$
\begin{aligned}
Z_{1,j} &:= \sum_{j=1}^{N} p_{jk} F_{0,j} \overline{V}_j F_{0,j}^T + \sum_{j=1}^{N} p_{jk} \pi_{0,j} G_{0,j} G_{0,j}^T, \\
K_{0,j} \Lambda_{0,j} K_{0,j}^T &:= Z_{1,j} - \overline{\Psi}_0 Z_{0,j} \overline{\Psi}_0^T, \\
V_{0,j} \Omega_{0,j} V_{0,j}^T &:= \pi_{1,j} - \pi_{0,j}.
\end{aligned} \tag{18}
$$

Step 1 **-** *Compute $K_{i+1,k}$ using a unitary matrix Σ_i of appropriated dimensions*

$$
\begin{bmatrix} \mathscr{L}_1 & \mathscr{M}_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \mathscr{L}_2 & \mathscr{M}_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \mathscr{L}_N & \mathscr{M}_N \end{bmatrix} \Sigma_i := \begin{bmatrix} K_{i+1,1} & 0 & \dots & 0 & 0 \\ 0 & K_{i+1,2} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & K_{i+1,N} & 0 \end{bmatrix}. \tag{19}
$$

where

$$
\begin{aligned}
\mathscr{L}_k &:= \begin{bmatrix} L_{1,k} & L_{2,k} & \dots & L_{N,k} \end{bmatrix}, && (20) \\
\mathscr{M}_k &:= \begin{bmatrix} M_{1,k} & M_{2,k} & \dots & M_{N,k} \end{bmatrix}, && (21) \\
L_{i,k} &:= \begin{bmatrix} p_{1k}^{1/2} F_{i,1} K_{i,1} & \dots & p_{Nk}^{1/2} F_{i,N} K_{i,N} \end{bmatrix}, && (22) \\
M_{i,k} &:= \begin{bmatrix} p_{1k}^{1/2} G_{i,1} V_{i,1} & \dots & p_{Nk}^{1/2} G_{i,N} V_{i,N} \end{bmatrix}, && (23) \\
\overline{\Lambda}_{i,k} &:= diag(\Lambda_{i,k}), && (24) \\
\overline{\Omega}_{i,k} &:= diag(\Omega_{i,k}), && (25) \\
K_{i+1,k} \Lambda_{i+1,k} K_{i+1,k}^T &:= Z_{i+2,k} - \overline{\Psi}_{i+1} Z_{i+1,k} \overline{\Psi}_{i+1}^T, && (26) \\
V_{i,j} \Omega_{i,j} V_{i,j}^T &:= \pi_{i+1,j} - \pi_{i,j}, \quad j,k = 1,\dots,N && (27)
\end{aligned}
$$

where $\Lambda_{i,k} \in \Re^{\alpha \times \alpha}$ e $\Omega_{i,k} \in \Re^{\beta \times \beta}$ are signature matrices, and $K_{i+1,k} \in \Re^{n \times \alpha}$ and $V_{i,j} \in \Re^{1 \times \beta}$.

Step 2 **-** *If desired, $Z_{i+2,k}$ can be computed as*

$$
Z_{i+2,k} := \overline{\Psi}_{i+1} Z_{i+1,k} \overline{\Psi}_{i+1}^T + K_{i+1,k} \Lambda_{i+1,k} K_{i+1,k}^T. \tag{28}
$$

In the next result, we presented a fast array to compute Riccati equation (9).

Algorithm 4.2 *(Fast Array Algorithm for Riccati Equation)*

Step 0 **-** *Initial conditions:*

$$
\tilde{Z}_{0|-1} := \Pi_0,
$$

DOI: 10.21575/25254782rmetg2021vol6n41626

$$
\begin{aligned}
\tilde{Z}_{1|0} &:= \mathscr{F}_0\tilde{Z}_{1|0}\mathscr{F}_0^T - \mathscr{F}_0\tilde{Z}_{1|0}\mathscr{H}_0^T(\mathscr{H}_0\tilde{Z}_{1|0}\mathscr{H}_0^T + R_0)^{-1}\mathscr{H}_0\tilde{Z}_{1|0}\mathscr{F}_0^T + W_0,\\
L_0S_0L_0^T &:= \tilde{Z}_{1|0} - \Psi_0\tilde{Z}_{0|-1}\Psi_0^T,\\
V_0M_0V_0^T &:= R_1 - R_0,\\
Q_0E_0Q_0^T &:= diag\left[K_{1,k}\Lambda_{1,k}K_{1,k}^T\right] - \mathscr{F}_0\left(diag\left[K_{0,k}\Lambda_{0,k}K_{0,k}^T\right]\right)\mathscr{F}_0^T,\\
R_{e,0} &:= \mathscr{H}_0\tilde{Z}_{1|0}\mathscr{H}_0^T + R_0,\\
K_{p,0} &:= \mathscr{F}_0\tilde{Z}_{1|0}\mathscr{H}_0^T R_{e,0}^{-T/2}.
\end{aligned} \tag{29}
$$

Step 1 **-** *Compute L_{i+1} using a $(I \oplus S_i \oplus M_i \oplus E_i \oplus)$-unitary matrix Γ_i of appropriated dimensions*

$$
\begin{bmatrix} R_{e,i}^{1/2} & \mathscr{H}_{i+1}L_i & V_i & 0 \\ \Psi_{i+1}K_{p,i} & \mathscr{F}_{i+1}L_i & 0 & Q_i \end{bmatrix}\Gamma_i := \begin{bmatrix} R_{e,i+1}^{1/2} & 0 & 0 & 0 \\ K_{p,i+1} & L_{i+1} & 0 & 0 \end{bmatrix} \tag{30}
$$

where

$$
\begin{aligned}
L_iS_iL_i^T &:= \tilde{Z}_{i+1|i} - \Psi_i\tilde{Z}_{i|i-1}\Psi_i^T,\\
V_iM_iV_i^T &:= R_{i+1} - R_i,\\
Q_iE_iQ_i^T &:= diag\left[K_{i+1,k}\Lambda_{i+1,k}K_{i+1,k}^T\right] - \mathscr{F}_i\left(diag\left[K_{i,k}\Lambda_{i,k}K_{i,k}^T\right]\right)\mathscr{F}_i^T,\\
R_{e,i} &:= \mathscr{H}_i\tilde{Z}_{i|i-1}\mathscr{H}_i^T + R_i,\\
K_{p,i} &:= \mathscr{F}_i\tilde{Z}_{i|i-1}\mathscr{H}_i^T R_{e,i}^{-T/2}
\end{aligned} \tag{31}
$$

where $S_i \in \Re^{\alpha_1\times\alpha_1}$, $M_i \in \Re^{\alpha_2\times\alpha_2}$ and $E_i \in \Re^{\alpha_3\times\alpha_3}$ are signature matrices; and $L_i \in \Re^{Nn\times\alpha_1}$, $V_i \in \Re^{m\times\alpha_2}$ and $Q_i \in \Re^{Nn\times\alpha_3}$.

Step 2 **-** *If desired, $\tilde{Z}_{i+2|i+1}$ can be computed as*

$$
\tilde{Z}_{i+2|i+1} := \Psi_{i+1}\tilde{Z}_{i+1|i}\Psi_{i+1}^T + L_{i+1}S_{i+1}L_{i+1}^T. \tag{32}
$$

Remark 4.1 *Note that in the fast array algorithm proposed is not necessary the hypothesis of stationarity of the Markov chain distribution to assure that W and $\mathscr{D}$ are constant matrices as in (TERRA; ISHIHARA; JESUS, 2012).*

Remark 4.2 *Note that for the case when there are no jumps in (4), $(N = 1)$, the fast array algorithm (30) collapses to the fast array algorithm for state-space systems proposed in (SAYED; KAILATH, 1994).*

Remark 4.3 *In order to quantify the number of operations per iteration required to implement the fast array algorithm proposed, consider that the rank of $\delta\tilde{Z}_i = \tilde{Z}_{i+2|i+1} - \tilde{Z}_{i+1|i}$, $\delta R_i =$*

$R_{i+2|i+1} - R_{i+1|i}$ and $\delta W_i = diag\left[K_{i+1,k}\Lambda_{i+1,k}K_{i+1,k}^T\right] - \mathscr{F}_{i+1} \times \left(diag\left[K_{i,k}\Lambda_{i,k}K_{i,k}^T\right]\right)\mathscr{F}_{i+1}^T$ are given by α_1, α_2, α_3 for all $i \geq 0$ and that $\alpha_1 \leq Nn$, $\alpha_2 \leq m$ and $\alpha_3 \leq Nn$ respectively. The dimension of the pre-array of (30) is defined by $(m+Nn)\times(m+\alpha_1+\alpha_2+\alpha_3)$. If compared with the pre-array of the algorithm proposed in (TERRA; ISHIHARA; JESUS, 2009), whose dimension is given by $(m+Nn)\times(m+3Nn)$, the reduction of the operations required is clear.

5 An Application to the RLS Problem

In this section, we developed a numerical example for the fast array algorithms based on a special case of structured time-variant parameter that arises in the recursive least-squares problem in adaptive filtering. The case for state-space models was first presented in (SAYED; KAILATH, 1994), here we develop the case for MJLS.

The problem described in (SAYED; KAILATH, 1994) reads as follows: given pairs of data points $\{u_i, d(i)\}$, $i = 0, 1, \ldots, N$, where u_i consists of the values of M input channels at time i, we are required to determine the linear least-squares estimate of an vector of unknown tap weights, w, so as to minimize the exponentially weighted error sum

$$\varepsilon = (w-\overline{w})^T \Pi_0^{-1}(w-\overline{w}) + \sum_{i=0}^{n} \lambda^{N-i}|d(i)-u_i w|^2, \tag{33}$$

where the parameter λ is often called the forgetting factor, since past inputs are exponentially weighted less than the more recent values. According to (SAYED; KAILATH, 1994), the problem (33) is equivalent to the following state-space estimation problem:

$$x_{i+1} = F_i x_i, \quad F_i = I_i\lambda^{-1/2}, \quad y_i = h_i x_i + v_i, \tag{34}$$

where x_i is the valued state sequence, $y_i = d_i/(\sqrt{\lambda})^i$ is the valued output sequence and v_i independent random variable with zero mean and covariance R_i.

In this case, the parameters F_i and h_i have a special structure viz., $h_i = h_{i+1}Z$ and $F_{i+1}Z = ZF_i$ where Z is the lower triangular shift matrix. Observe that with these relations the state-space model in (34) is a special structured time-variant model.

Now, in this paper, were considered that the parameters F_i and h_i are subject to Markovian jump. Therefore the state-space system (34) can be rewritten as the following MJLS:

$$x_{i+1} = F_{i,\Theta_i} x_i, \quad F_{i,\Theta_i} = I_{\Theta_i}\lambda^{-1/2}, \quad y_i = h_{i,\Theta_i} x_i + v_i, \tag{35}$$

where Θ_i is a time-discrete Markov chain with finite state space $\{1,\ldots,N\}$.

The system (35) can be rewritten in augmented form as

$$z_{i+1} = \mathscr{F}_i z_i, \quad y_i = \mathscr{H}_i z_i + \varphi_i, \tag{36}$$

where

$$z_i = \begin{bmatrix} z_{i,1} \\ \vdots \\ z_{i,N} \end{bmatrix}, \quad z_{i,k} = x_i 1_{\{\Theta_i = k\}}, \tag{37}$$

$$\mathscr{F}_i \ := \ \begin{bmatrix} p_{11}F_{i,1} & \dots & p_{N1}F_{i,N} \\ \vdots & \ddots & \vdots \\ p_{1N}F_{i,1} & \dots & p_{NN}F_{i,N} \end{bmatrix}, \ \mathscr{H}_i := \begin{bmatrix} h_{i,1} & \dots & h_{i,N} \end{bmatrix},$$

where

$$F_{i,j} = I_j \lambda^{-1/2}, \quad h_{i,j} = \ [u(i) \quad u(i-1) \quad \dots \quad u(0) \quad 0_{N-1}] \ , \quad \varphi_i = h_{i,\theta_i} v_i. \tag{38}$$

The filter for linear systems subject to Markovian jumps is given by the following equations

$$\hat{x}_{i+1|i} = \sum_{j=1}^{N} \hat{z}_{i+1,j|i}, \tag{39}$$

$$\hat{z}_{i+1|i} = \mathscr{F}_i \hat{z}_{i|i-1} + \mathscr{F} \tilde{Z}_{i|i-1} \mathscr{H}_i^T R_{\varepsilon,i}^{-1} (y(i) - \mathscr{H}_i \hat{z}_{i|i-1}), \tag{40}$$

$$R_{\varepsilon,i} = R_i + \mathscr{H}_i \tilde{Z}_{i|i-1} \mathscr{H}_i^T, \quad K_{p,i} = \mathscr{F} \tilde{Z}_{i|i-1} \mathscr{H}_i^T R_{\varepsilon,i}^{-T/2}. \tag{41}$$

The Riccati equation for linear systems subject to Markovian leaps can be written as

$$\tilde{Z}_{i+1|i} = \mathscr{F}_i \tilde{Z}_{i|i-1} \mathscr{F}_i^T - \mathscr{F}_i \tilde{Z}_{i|i-1} \mathscr{H}_i^T R_{\varepsilon,i}^{-1} \mathscr{H}_i \tilde{Z}_{i|i-1} \mathscr{F}_i^T + W, \tag{42}$$

where

$$R_i = \mathscr{D}_i \mathscr{D}_i^T, \quad \mathscr{D}_i = \ [\pi_{i,1}^{1/2} \quad \dots \quad \pi_{i,N}^{1/2}], \tag{43}$$

$$W_i = diag[Z_{i+1,k}] - \mathscr{F}_i (diag[Z_{i,k}]) \mathscr{F}_i^T, \tag{44}$$

$$Z_{i+1,k} = \sum_{j=1}^{N} p_{jk} F_{i,j} Z_{i,j} F_{i,j}^T. \tag{45}$$

The variation of the parameters is done by a Ψ_i, $\Re^{n \times n}$ convenient, $\Psi_i = diag[\overline{\Psi}_{i,k}]$.

Applying this matrix in the difference equation of the filter we obtain

$$\tilde{Z}_{i+1|i} - \Psi_i \tilde{Z}_{i|i-1} \Psi_i^T \equiv L_i S_i L_i^T. \tag{46}$$

Now consider the parameters varying according to the following rules

$$\mathscr{H}_{i+1}\Psi_i = \mathscr{H}_i, \quad H_{i,k} = H_{i+1,k}\overline{\Psi}_{i,k}, \tag{47}$$

$$\mathscr{F}_{i+1}\Psi_i = \Psi_{i+1}\mathscr{F}_i, \quad \overline{\Psi}_{i+1,k}F_{i,k} = F_{i+1,k}\overline{\Psi}_{i,k}. \tag{48}$$

The Riccati equation (42) can be calculated alternatively by the following fast array algorithm

$$\begin{bmatrix} R_{\varepsilon,i}^{1/2} & \mathscr{H}_{i+1}L_i & V_i & 0 \\ \Psi_{i+1}K_{p,i} & \mathscr{F}_{i+1}L_i & 0 & Q_i \end{bmatrix} \Gamma_i = \begin{bmatrix} R_{\varepsilon,i+1}^{1/2} & 0 \\ K_{p,i+1} & L_{i+1} \end{bmatrix}. \tag{49}$$

where

$$Q_i E_i Q_i = diag\left[K_{i+1,k}\Lambda_{i+1,k}K_{i+1,k}^T\right] - \mathscr{F}_{i+1}\left(diag\left[K_{i,k}\Lambda_{i,k}K_{i,k}^T\right]\right)\mathscr{F}_{i+1}^T, \tag{50}$$

$$K_{i,k}\Lambda_{i,k}K_{i,k}^T = Z_{i+1,k} - \overline{\Psi}_{i,k}Z_{i,k}\overline{\Psi}_{i,k}^T, \tag{51}$$

$$V_i N_i V_i^T = R_{i+1} - R_i. \tag{52}$$

The difference Lyapunov (51) can be calculated by the extended fast array algorithm $(\mathscr{L}_i\Sigma_i = \mathscr{K}_{i+1})$ dado por

$$\begin{bmatrix} L_{i,1} & 0 & \cdots & 0 \\ 0 & L_{i,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{i,N} \end{bmatrix} \Sigma_i = \begin{bmatrix} K_{i+1,1} & 0 & \cdots & 0 \\ 0 & K_{i+1,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{i+1,N} \end{bmatrix}, \tag{53}$$

where

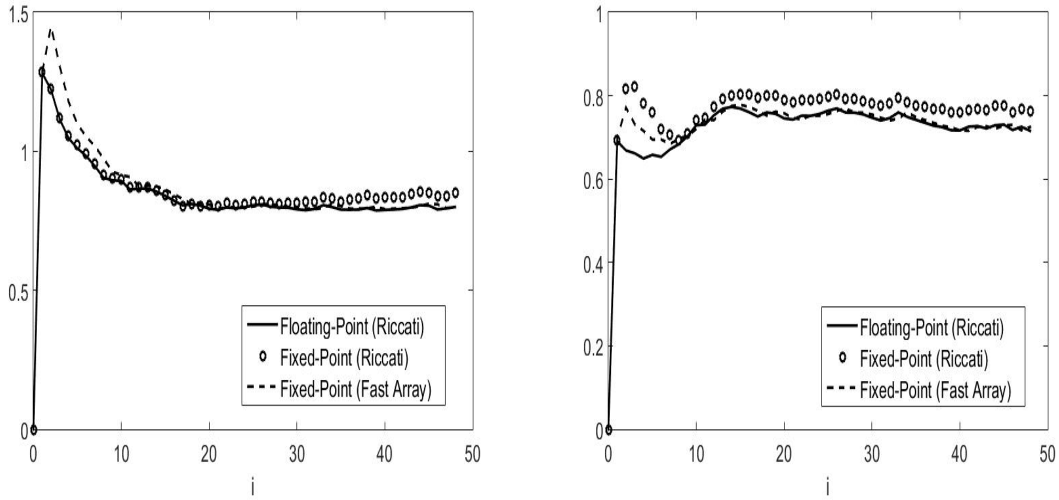
$$L_{i,k} = \begin{bmatrix} p_{1k}^{1/2}F_{i+1,1}K_{i,1} & p_{2k}^{1/2}F_{i+1,2}K_{i,2} & \cdots & p_{Nk}^{1/2}F_{i+1,N}K_{i,N} \end{bmatrix}. \tag{54}$$

The parameters used for this application are as follows:

$$F_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, F_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, H_1 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}^T,$$
$$H_2 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}^T, \pi = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}^T, P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}.$$

DOI: 10.21575/25254782rmetg2021vol6n41626

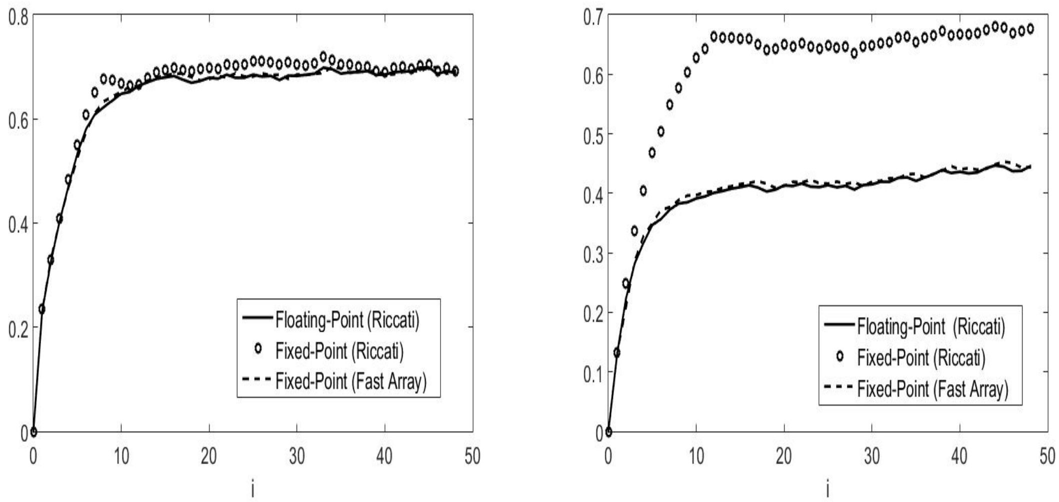
Figura 1: First and Second Singular Values of $\tilde{Z}_{i|i-1}$.



(a) First Singular Value (b) Second Singular Value

Source: Produced by the Author.

Figura 2: Third and Fourth Singular Values of $\tilde{Z}_{i|i-1}$.



(a) Third Singular Value (b) Fourth Singular Value

Source: Produced by the Author.

We present in the following two examples to show some features of the fast array algorithm proposed. In the first example, we computed the singular values of $\tilde{Z}_{i|i-1}$ for three different implementations: floating-point, fixed-point for explicit Riccati equation and for fast *array* algorithm. It was utilized a fixed-point architecture with 16-bit architecture where the range of the computation was defined in the interval of $-65,543$ to $65,543$. The Riccati equation on the basis of floating-point architecture was used as a reference because we want to

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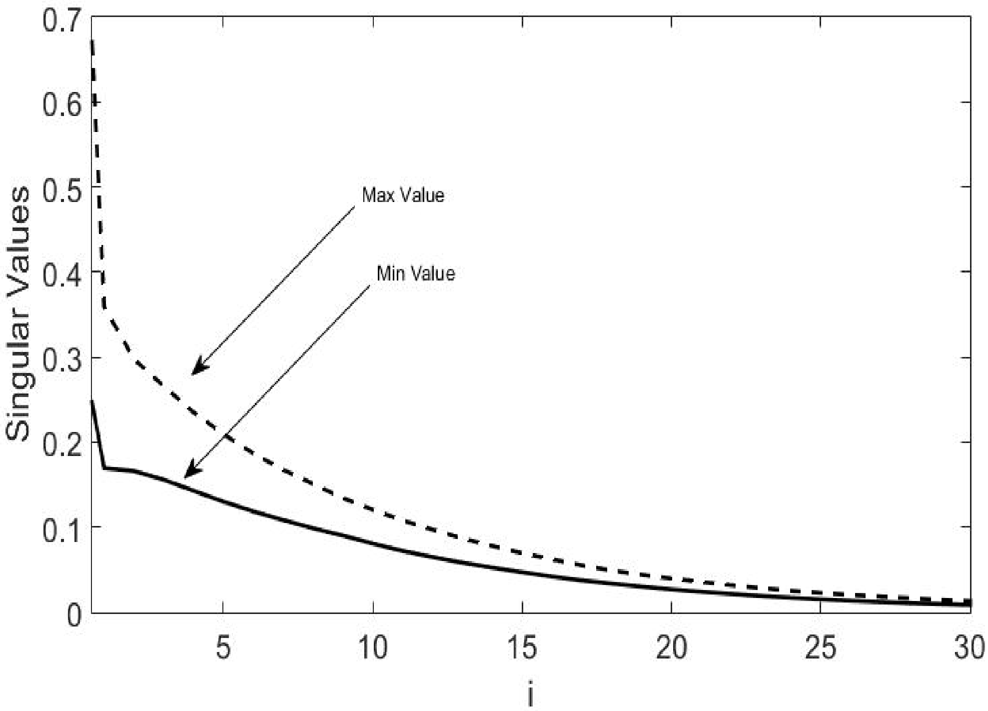
Figura 3: Singular Values of L_i



Source: Produced by the Author

test the reduction of the dynamic range of the computation of these two methods in fixed-point architecture. Note that the performance of the fast array algorithm in fixed-point implementation is closer to the performance of the Riccati equation in floating-point implementation as can be seen in Figures (1) and (2). In the second example, we evaluated the matrix L_i. The minimum and maximum singular values of L_i go to zero in 30 iterations approximately as can be seen in Figure (3), that is, there is a loss of rank, which makes the algorithm faster.

6 Conclusion

In this paper was developed the fast array algorithms for the LMMSE for a class of MJLS with structured time-variant parameters. It assumes an important role in the computational of this estimator, because of the dimensions of the parameters matrices related with the augmented model of MJLS. The numerical examples illustrate the importance of this approach.

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Enviado em: 09 jun. 2021.

Aceito em: 02 dez. 2021.

Editor responsável: Mateus das Neves Gomes.

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